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# Observation of conductance oscillations in a superconducting Andreev diffractometer

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## Abstract

We observed superconductivity-induced conductance oscillations, arising from quasiparticle interference effects in a single normal-metallic (N) electrode of finite width in contact with a superconductor (S). A  $10 \times 10 \mu\text{m}^2$  Al patch was overlaid on a pre-evaporated 0.5- $\mu\text{m}$ -wide mesoscopic silver wire. A phase gradient along the N/S interface was induced by applying a magnetic field perpendicular to the plane of the superconducting Al patch. The conductance across the N/S interface showed Fraunhofer-like oscillations with amplitudes of a small fraction of  $2e^2/h$  per conducting channel, for fields up to 450 G. For sufficiently low fields and temperatures the interfacial diffraction effect of quasiparticles is more complicated due to the field-induced electron–hole dephasing effect. None the less, the overall feature of the oscillatory magnetoconductance from the N/S interface appears to confirm the diffraction of Andreev-reflected quasiparticles proposed theoretically for a single N/S interface with a phase gradient. © 2001 Elsevier Science B.V. All rights reserved.

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Keywords: Andreev diffractometer; Andreev reflection; Normal-metal/superconductor interface; Phase coherence

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## 1. Introduction

Spivak and Khmelnitskii [1] have pointed out that a phase-coherent normal-metallic electrode (N) in contact with two superconductors (S) shows an oscillatory conductance as a function of the

phase difference  $\Delta\phi = \phi_2 - \phi_1$  between the order parameters of two superconductors. When electrons in N are reflected at an N/S interface some electrons, as a result of the Andreev reflection [2], transform into holes, while acquiring the local phase of the order parameter of the comprising superconductor. Other incident electrons are normally reflected as electrons without a change in their phases upon reflection. Two partial waves from a single incident electron, when Andreev reflected at two different N/S interfaces and consequently recombined, thus, acquire an addendum phase difference  $\phi_2 - \phi_1$ . Consequent conductance

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oscillations have been observed [3–5] in phase coherent mesoscopic systems containing N/S interfaces as the phase difference of the two superconductors was varied by the application of an external current or a magnetic field.

On the other hand, Cook et al. [6] have suggested that the conductance of a normal metallic electrode in contact with a single superconductor, possessing a phase gradient of the order parameter along the N/S interface, should also be oscillatory with Fraunhofer-like patterns, which is similar to optical diffraction from a single aperture. The authors assume a phase-coherent mesoscopic system which is connected to external current-carrying leads. The conductance is calculated by considering the quantum-mechanical scattering of excitations of energy  $E$  for all incoming and outgoing scattering channels and constructing the corresponding matrix of reflection and transmission coefficients. The calculation result indicates that, in the limit of negligible quasiparticle transmissions as in the case of a superconductor much longer than its coherence length, the conductance reduces to [7,8]  $G = (2e^2/h)[2R_a R'_a / (R_a + R'_a)]$ . Here,  $R_a$  and  $R'_a$  are probabilities for Andreev reflection of quasiparticles incident from the left and right electrodes of Fig. 1, respectively. For a system with clean N/S interfaces, i.e., without an insulating barrier I in Fig. 1, the situation is symmetric and the conductance is further simplified to  $G = (2e^2/h)R_a$ .

Solving the Bogoliubov–de Gennes equation with a two-dimensional tight-binding system, the conductance can be shown to oscillate as the phase gradient of the order parameter of the supercon-

ductor is varied [7,8]. But, no conductance oscillations occur for a system with perfectly clean N/S interfaces. In this case, it is argued [7,8] that predominant Andreev reflections lead to the approximate sum rule as  $R_a$  being the total number of open channels,  $M$ , for any phase gradients. For a system with a non-ideal N/S interface as illustrated by the presence of an insulating barrier I at an N/S interface as in Fig. 1, the zero-temperature conductance is expected to oscillate with Fraunhofer-like patterns as the phase gradient varies, where the sum rule is broken by the normal reflection at the interface. The amplitude of conductance oscillations per conducting channel is expected to be a small fraction of the conductance of a wire of perfect transparency,  $2e^2/h$ , with the periods of  $2\pi$  phase difference along the interface.

## 2. Experiment

In order to observe the conductance oscillations, we adopted the geometry as illustrated in Fig. 2. In this geometry three normal-metallic electrodes were deposited first and a rectangular superconducting patch was overlaid. The current  $I$  was injected from the left N electrode to the right one via the patch and the voltage difference  $V$  between the leads  $V_1$  and  $V_2$  was measured. The distance between the voltage lead of the left N and the superconducting patch is  $L_1$ , while the portion of length  $L_2$  underneath the superconducting patch provides an N/S interface. When the patch becomes superconducting, the wave function of the order parameter is written as  $\Psi(\vec{r}, t) = |\Psi(\vec{r}, t)| e^{i\theta(\vec{r}, t)}$ .  $|\Psi(\vec{r}, t)|$  and  $\theta(\vec{r}, t)$  are the amplitude and the phase of the superconducting order, respectively, where the phase gradient  $\nabla\theta(\vec{r}, t)$  is given by

$$\nabla\theta(\vec{r}, t) = \frac{2\pi}{\Phi_0} [\vec{A}(\vec{r}, t) + \lambda J_s(\vec{r}, t)]. \quad (1)$$

Here,  $\vec{A}(\vec{r}, t)$  is the vector potential,  $J_s(\vec{r}, t)$  is the supercurrent density inside the superconducting patch, and  $\lambda = m/ne^2 = \mu_0\lambda^2 [n(\equiv |\Psi(\vec{r}, t)|^2)]$ ,  $e$ , and  $m$  are the density, the charge, and the effective mass of an electron, respectively,  $\lambda$  is the penetration depth, and  $\mu_0$  is the free-space permeability]. Thus, the phase of the order parameter in the

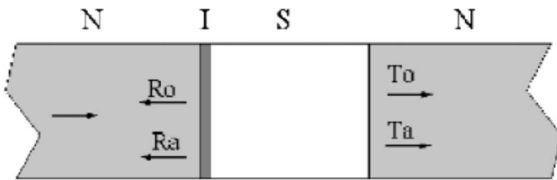


Fig. 1. Schematic configurations of an Andreev diffractometer adopted in Ref. [6] to calculate the conductance. Layer I denotes the scattering barrier due to any interfacial imperfection.  $R_0$  and  $T_0$  ( $R_a$  and  $T_a$ ) are the probabilities of normal (Andreev) reflection and transmission of quasiparticles incident from the left normal (N) electrode.

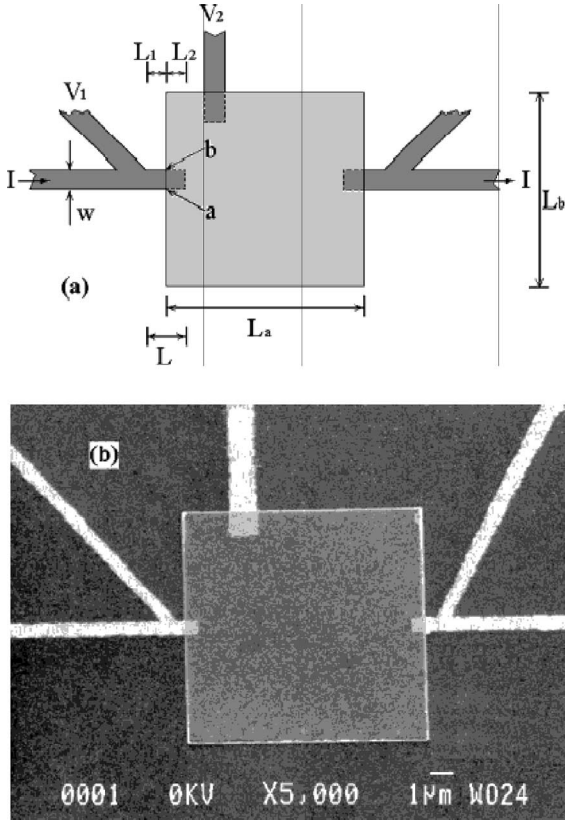


Fig. 2. (a) Schematic configuration and (b) SEM picture of the sample used in this experiment. All normal electrodes are made of thermally deposited Ag and the overlaid square superconducting patch is made of Al.

superconductor can be controlled by the application of an external current or a magnetic field. The total phase difference  $\Delta\theta$  along the N/S interface for the application of an external magnetic field is

$$\Delta\theta = \int_a^b \nabla\theta d\vec{r} = 2\pi \frac{\Phi}{\Phi_0} \frac{W}{2(L_a + L_b)}, \quad (2)$$

where  $\Phi_0$  ( $\equiv h/2e$ ) is the flux quantum and  $\Phi$  ( $\equiv HL_aL_b$ ) is the magnetic flux excluded by the square superconducting patch with the lateral dimensions of  $L_a$  and  $L_b$ . Thus,  $\Delta\theta$  is determined by the total excluded flux and the ratio between the width of the interface and the circumference of the superconducting patch,  $2(L_a + L_b)$ . In Eq. (2), we ignored the edge effect at the perimeter of the su-

perconducting patch. For  $L_a = L_b = 10 \mu\text{m}$  and the width of  $W = 0.5 \mu\text{m}$  (Fig. 2(a)), a variation of magnetic field of  $\Delta H \approx 17 \text{ G}$  is required to generate a total phase difference of  $2\pi$  along the interface.

The structure of a sample was made using the combination of electron-beam lithography, Ar-ion etching, and lift-off techniques. We used silicon substrate covered with the native oxide layer. The normal-metallic electrodes were made by evaporating an 80-nm-thick Ag layer on the patterned resist and lifting off subsequently to the width of  $W = 500 \text{ nm}$  (the vertical one was a little wider). Then, a 100-nm-thick Al layer was deposited as the superconducting patch on the second patterned resist and lifted off, leaving an Al layer in the area of  $10 \times 10 \mu\text{m}^2$  (refer to Fig. 2). The surface of the N electrodes was cleaned using ion milling right before the Al deposition to enhance the transparency of the Ag/Al interface. Both  $L_1$  and  $L_2$  were set to be  $0.5 \mu\text{m}$ .

### 3. Results and discussion

Data were taken by the four-probe lock-in technique at 38 Hz. The resistance of 7- $\mu\text{m}$ -long portion of Ag wire (not shown in Fig. 1) was  $4.0 \Omega$ , corresponding to the resistivity of  $2.3 \times 10^{-6} \Omega\text{cm}$ . Using the value of  $\rho l = 5.36 \times 10^{-12} \Omega\text{cm}^2$  for Ag [9], we get  $l = 2.3 \times 10^{-6} \text{ cm}$  and the diffusion coefficient of electrons  $D = (1/3)v_F l \approx 108 \text{ cm}^2/\text{s}$ . The corresponding electron-hole (e-h) phase coherence length is  $\xi = (\hbar D / 2\pi k_B T)^{1/2} \approx 0.32 \mu\text{m}$  at  $T = 0.13 \text{ K}$ . The single-particle phase breaking length  $L_\phi = (D\tau_\phi)^{1/2}$ , where  $1/\tau_\phi$  is the sum of the phase breaking rates, is estimated to be about [3–5]  $1.7 \mu\text{m}$  for silver at  $T = 0.1 \text{ K}$ , which is larger than  $L_1$ .

The resistance vs temperature for the Ag/Al hybrid structure is shown in Fig. 3. The bias current used was  $0.3 \mu\text{A}$ . In the figure the critical temperature  $T_c$  of the aluminum patch is about  $1.64 \text{ K}$ , which is somewhat enhanced above the pure bulk value due to the disorder effect. With lowering temperature below  $T_c$  the resistance keeps dropping, reaches a minimum, and shows a slight increase in the lowest temperature range used. The

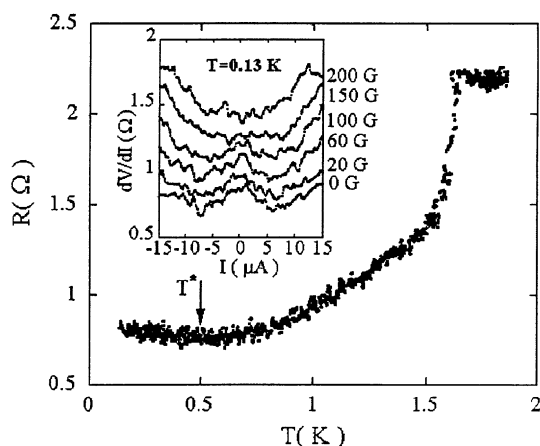


Fig. 3. The resistance vs temperature taken in a bias current of  $0.3 \mu\text{A}$ . The superconducting transition temperature  $T_c$  for Al was about  $1.64 \text{ K}$  and the temperature of the minimum resistance was about  $0.5 \text{ K}$ . Inset: the differential conductance  $dV/dI$  as a function of the applied dc bias current at  $T = 0.13 \text{ K}$  for varied magnetic fields. For the sake of clarity, each set of the differential conductance in an external field is shifted by  $0.1 \Omega$  from the neighboring set below.

temperature  $T^*$  which gives the resistance minimum is about  $0.5 \text{ K}$ . According to a quasiclassical theory this value should correspond to  $5\epsilon_{\text{Th}}/k_B$ , where  $\epsilon_{\text{Th}} = \hbar D/L^2$  is the Thouless energy [10,11] for the N electrode of length  $L$ . The calculated value of  $T^* \simeq 0.42 \text{ K}$  corresponding to  $\epsilon_{\text{Th}} = 7.0 \mu\text{eV}$  is consistent with the observed value of  $0.5 \text{ K}$ . In the temperature regime of conductance reentrance [10,11] ( $T \leq T^*$ ), where the phase coherence length of quasiparticles in N is longer than the length  $L$  of the N electrode, the resistance increases with decreasing temperature. In the regime of the classical proximity effect ( $T \geq T^*$ ), however, the resistance decreases with decreasing temperature. The existence of the slight reentrance of resistance indicates that the left Ag/Al interface is not highly transparent, but it is transparent enough so as to allow the Andreev reflection not to be negligible comparing to the specular reflection. We could not obtain quantitative estimate of the transparency, however, because the sole junction resistance was not obtainable from the geometry adopted in this study.

The inset of Fig. 3 shows the differential resistance  $dV/dI$  vs dc bias current for varied magnetic

fields at  $T = 0.13 \text{ K}$ . An ac current modulation of  $0.3 \mu\text{A}$  was used for the measurement. The differential resistance in the inset reveals large aperiodic oscillations. The oscillations are random fluctuations which are not related with the particular sample geometry. In zero field, the bias current,  $I^*$ , corresponding to the onset temperature of the resistance reentrance,  $T^*$ , is about  $8 \mu\text{A}$ , which is equivalent to the bias voltage of about  $7 \mu\text{eV}$  at  $T = 0.13 \text{ K}$ . This value of  $I^*$  is consistent with the slight enhancement of the differential resistance for  $I \lesssim 8 \mu\text{A}$  in the inset of Fig. 3. The zero-bias enhancement of the differential resistance disappears around  $150 \text{ G}$  and above. This value is close to the phase-breaking field between the electrons and the Andreev-reflected holes [12],  $H_c = h/(eL_1W) = 160 \text{ G}$ , which corresponds to one flux quantum threading the normal-metal region of width  $W$  and length  $L_1$  between a voltage lead and the superconductor. In a field above  $H_c$  the zero-bias resistance enhancement disappears as an electron and the Andreev-reflected conjugate hole lose phase coherence between them.

The conductance vs magnetic field for the Ag/Al structure is shown in Fig. 4 for varied temperatures from  $0.13$  to  $2.95 \text{ K}$ . The bias current was  $1 \mu\text{A}$ . Although not shown, the observed conductance at all the temperatures used was symmetric

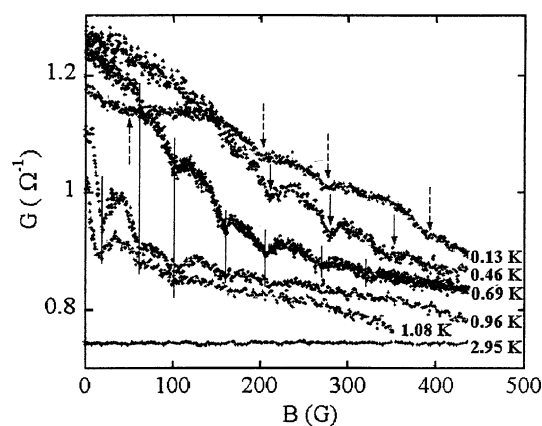


Fig. 4. The conductance oscillations as a function of the applied magnetic field, taken in a bias current of  $1 \mu\text{A}$  for temperatures  $0.13$ – $2.95 \text{ K}$ . The conductance for  $T = 2.95 \text{ K}$  is shifted upward by  $0.3 \Omega$ .

to the one for negative fields. At temperatures below the superconducting transition ( $T_c = 1.64$  K) of Al the conductance shows nontrivial oscillatory features, although the envelope of the conductance keeps decreasing over the whole measured field range. At temperatures above  $T_c$ , however, the conductance is insensitive to the field change as shown in Fig. 4 for  $T = 2.95$  K. Thus the oscillatory features of the conductance in Fig. 4 is clearly related to the Al patch of being superconducting.

At  $T = 0.13$  K, which is in the temperature regime of resistance reentrance, the conductance shows clear oscillations for fields up to about 400 G (see the dashed arrows in Fig. 4 for the local minima). The first broad minimum takes place at  $H \approx 50$  G. At  $T = 0.46$  K, a temperature slightly above  $T^*$ , the conductance shows a monotonic decrease for a field up to  $H \approx 200$  G, above which at least two oscillations with a period of  $\sim 65$  G are visible (see the solid arrows in Fig. 4 for the local minima). As the temperature further increases the conductance oscillations become more apparent, with the decreased field of the first minimum and the field period. The first conductance minimum occurs at  $\sim 65$  and  $\sim 20$  G for  $T = 0.69$  and  $0.96$  K ( $1.08$  K as well), respectively, with the corresponding averaged field periods of  $50$ – $55$  G. At  $T = 0.96$  K, a temperature well in the classical proximity effect regime, the conductance oscillations become most conspicuous. Above  $0.96$  K the oscillations become less clear and disappear completely at temperatures above  $T_c$  of the Al patch. We also notice that, above  $T^*$ , the field period of the oscillations is almost independent of temperature. Except for the primary oscillation near the zero field, the overall feature of the conductance curve, where the oscillation period becomes more regular and the amplitudes of the local maxima keep decreasing with increasing fields, is reminiscent of the Fraunhofer pattern. For no clear reasons, the first conductance minimum occurs at about one half of the field period of higher-order conductance modulations for  $T = 0.96$  and  $1.08$  K. This is the major discrepancy from the expected Fraunhofer-like behavior. In general, as the temperature decreases the last oscillation survives for a higher magnetic field, presumably because the

phase coherence length of quasiparticles in N increases.

For the sample geometry shown in Fig. 2 an external magnetic field can affect the conductance in two different ways. First, the conductance changes due to the e–h dephasing effect by an external magnetic field, as discussed above in relation with the inset of Fig. 3. This effect takes place in the magnetoconductance of an N/S interface *even without any phase gradient along the interface*. This gives a rather abrupt conductance change for a field up to  $H_c$  as observed in our previous study (refer to Fig. 6(b) of Ref. [13]). For our sample,  $H_c$  is estimated to be about 160 G. The second effect of the magnetic field, which is expected to occur in our sample, is the diffraction of the Andreev-reflected quasiparticles from the N/S interface, as treated theoretically by Cook et al. [6] and discussed in detail above. The latter effect takes place most likely when the phase difference along the N/S interface is amplified by placing a large superconducting electrode as in this study. The estimated field period of the latter effect is about 17 G for our sample, but apparently the observed value  $50$ – $55$  G is significantly larger than the estimate. We believe that for a given external field below  $H_c$  both effects will appear at the same time. But the interfacial diffraction effect may be overwhelmed by the e–h dephasing effect in this low field range, especially for  $T < T^*$ , where the zero-field e–h phase coherence length exceeds the length of the normal electrode. We believe that is the reason for the difficulty of discerning Fraunhofer-like oscillations for low fields at  $0.13$  and  $0.46$  K. Especially, at  $0.13$  K, well inside the resistance-reentrance regime, the appreciable slow-down of the conductance decrease with increasing magnetic field in the range of  $50 < H < 200$  G is believed to be caused by the field-induced e–h dephasing effect. For a field above  $H_c$ , even at these low temperatures, the e–h coherence is already weakened significantly. In this case, the magnetoconductance is no longer highly affected by the e–h dephasing effect and consequently the interfacial diffraction effect becomes more conspicuous as seen in Fig. 4. As the temperature is raised above  $T^*$  the e–h phase coherence length is shortened to the scale less than the length of the N electrode. Thus, the

interfacial diffraction effect in this case also becomes conspicuous even for low fields, as the field-induced e–h dephasing effect is less effective.

The Fermi wavelength corresponding to  $n = 5.85 \times 10^{28} \text{ m}^{-3}$  for Ag is about 5 Å, which leads to the estimation for the number of open conducting channels,  $M \approx Wt/\lambda_F^2 \approx 1.6 \times 10^5$ , where  $t$  (= 800 Å) is the thickness of the Ag wire. The observed amplitude of the conductance oscillations per conducting channel turns out to be in the range of  $\sim 0.02\text{--}0.13 \times 2e^2/h$ , which is much smaller than the value of conductance for a perfectly transparent wire without an interface,  $2e^2/h$ . The value of the oscillation amplitude is close to what is estimated in Ref. [6] for reasonable interfacial parameters, but the quantitative comparison is irrelevant because the interfacial transparency is not available for our sample.

The periods of oscillations at any measurement temperatures below  $T_c$  turn out to be significantly larger than the estimated value  $\Delta H \approx 17 \text{ G}$  for  $L_a = L_b = 10 \text{ }\mu\text{m}$  and  $W = 0.5 \text{ }\mu\text{m}$ . In the calculation of Eq. (2) the Al patch is assumed to show perfect diamagnetism. Since the thin Al patch may act as a type-II superconductor the effective value of the excluded magnetic flux by the superconducting patch,  $\Phi$  in Eq. (2), can be significantly smaller than the naive estimation as adopted in the equation. The corresponding induction of the screening current at the rim of the superconducting patch is less than the case of the perfect diamagnetic exclusion of the magnetic flux, which makes the field period of Fraunhofer-like oscillations longer. In Fig. 4, we also observe that the oscillation period varies with field for temperatures of 0.13 and 0.46 K, which are below  $T^*$ , while above  $T^*$  the oscillation period is temperature independent. The mechanism of the temperature dependence of the oscillation period and its relevance to the onset temperature  $T^*$  are unclear.

As mentioned before, in deriving Eq. (2), the edge effect at the perimeter of the superconducting patch is ignored. Since the London penetration depth of Al, 160 nm, is shorter than the overlap distance  $L_2$  the phase gradient has a distribution near the edge of the superconducting patch; the phase gradient becomes smaller going into the

superconductor and vanishes beyond the penetration depth from the edge. This phase distribution near the contact region smears the diffraction pattern, but the period of the conductance oscillations is unaffected by the distribution of the phase gradient and depends only on the maximum phase gradient at the edge, which is determined by Eq. (2). Thus, the discrepancy of the observed field period of the conductance oscillations from the expectation is not due to the distribution of the phase gradient.

#### 4. Conclusion

We have observed clear conductance oscillations with Fraunhofer-like patterns in an Andreev diffractometer with a low-transparent N/S interface, especially, at temperatures above the minimum resistance,  $T^*$ . The amplitude of the conductance oscillations per conducting channel turns out to be in the range of  $0.02 - 0.13 \times 2e^2/h$ . This is consistent with the theoretical prediction by Cook et al. [6]. However, the field period of the oscillations is significantly longer than the prediction, presumably because of the ineffective field shielding by the superconducting film due to its type-II nature. In the low-field range below  $H_c$  and for temperatures below  $T^*$ , the e–h dephasing due to an external magnetic field dominates the interfacial diffraction effect, which results in the significant slow-down of the conductance decrease with increasing external magnetic field up to 200 G. The conductance oscillations disappear completely for temperatures above the superconducting transition of Al film, which confirms that the oscillations observed in this experiment stems from the diffraction from the N/S normal-metal/superconductor interface.

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